

POSSIBLE HIGGS BOSON EFFECTS ON THE RUNNING OF THIRD AND FOURTH GENERATION QUARK MASSES AND MIXINGS

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ABSTRACT

The Schwinger-Dyson equation for the quark self-energy is solved for the case of the third and fourth quark generations. The exchanges of standard model gluons and Higgs bosons are taken into account. It is found that Higgs boson exchange dominates the quark self-energy in the ultraviolet region for sufficiently large input quark masses causing the running quark propagator mass to increase with energy-scale. No running of the quark mixing angles is found for input quark masses up to and including 500 GeV.

Unlike the renormalization group equation, the Schwinger-Dyson equation (SDE) analysis presented here enables us in principle to calculate the dynamical quark mass functions for very light quarks at low energies where quantum chromodynamics (QCD) is in the non-perturbative region. This is of interest because experimentally accessible quantities are presently found at low energy-scales. In the presence of heavy quarks, such as bottom and top, it allows the calculation of running mass matrix at momentum scale of the order or less than the quark masses, at which scale the RG equation results may not be reliable. Moreover, this nonperturbative SDE approach is also interesting in that it can describe the running mass function of an ultra-heavy quark, with a large Yukawa coupling that cannot be treated reliably by perturbation theory. This would be relevant for a fourth generation quark.

In this study we assume that chiral symmetry is broken and solve the quark propagator SDE to find running quark mass functions and running quark mixing angles. In addition to the usual gluon contributions, we include Higgs boson exchange effects in the quark self-energy. The contributions to the quark self-energy due to Higgs bosons are being studied here for the first time in the context of the quark propagator SDE. We also study for the first time in a Higgs-boson-plus-gluon model the multigenerational cases of quarks which enable us to analyze quark mixing angles.

The general form of the quark propagator SDE that we use is

$$S^{-1}(q) = S_0^{-1}(q) - \int \frac{d^4k}{(2\pi)^4} g_s \gamma_\mu S(k) g_s \Lambda_\nu G^{\mu\nu} - \int \frac{d^4k}{(2\pi)^4} g_H S(k) g_Y \Lambda_H P_H, \quad (1)$$

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where $g_s\gamma_\mu$ and $g_s\Lambda_\nu$ are quark-gluon vertices, $S(k)$ is the quark propagator, $G^{\mu\nu}$ is the gluon propagator, $g_Y\Lambda_H$ is the quark-Higgs boson vertex factor, and P_H is the Higgs boson propagator.

The ladder approximation quark propagator in the Landau gauge in Minkowski space that we use is $iS(k) = (\not{k} + M)(k^2 - M^2)^{-1}$, where M is a 2×2 quark mass function matrix, assumed to be symmetric for this study. The gluon propagator we use is

$$g_s^2 G^{\mu\nu}(k) \equiv i \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) G(k), \quad \text{with } G(k) = \frac{g_s^2(k)}{k^2} \quad (2)$$

and the leading-log (one-loop) coupling

$$g_s^2(k) = \frac{4\pi^2 d}{\ln(x_0 - \frac{k^2}{\Lambda_{QCD}^2})} \quad (3)$$

where $d = 12/(33 - 2n_f)$, n_f is the number of flavors. This leading-log QCD coupling is renormalization-scheme and gauge independent, as is its second order, two-loop, extension ¹. x_0 is a parameter that functions as a smooth infrared cutoff. Eq. (3) does not accurately model the gluon potential in the infrared region, but it does enable us to assess the relative magnitude and shape effects of the Higgs boson and gluon exchanges in the quark self-energy ³.

For the Yukawa sector we consider a 2×2 symmetric Yukawa coupling matrix, which can be diagonalized by a 2×2 unitary matrix, to give diagonal elements g_\pm . The one-loop Renormalization Group Equation for the diagonalized Yukawa couplings, g_\pm , without weak interaction effects, is

$$\frac{dg_\pm}{dt} = \frac{1}{4\pi^2} \left(\frac{9}{8} g_\pm^3(t) - 2g_{QCD}^2(t)g_\pm(t) \right) \quad (4)$$

with $t = \ln(x_0 + q^2/\Lambda_{QCD}^2)/2$, and $\Lambda_{QCD} = 0.18$ GeV. This has solution

$$g_\pm^2 = \frac{1}{C_\pm t^{2d} - \frac{9}{16\pi^2} \frac{t}{1-2d}} \quad (5)$$

where the C_\pm must be determined by boundary conditions. We apply the boundary conditions $m_\pm^2(1 \text{ GeV}) = g_\pm^2(1 \text{ GeV})v^2/2$ for small (< 150 GeV) running quark masses, and $m_\pm^2(m_\pm) = g_\pm^2(m_\pm)v^2/2$ for large running quark masses, where $v \simeq 246$ GeV.

We solve the resulting SDE by converting it into a second order differential equation and applying a fourth-order Runge-Kutta subroutine ². We input $M_{bottom}(0) = 4.89$ GeV, $M_{top}(0) = 179$ GeV, and for the degenerate fourth generation quarks $M_{bottom'}(0) = M_{top'}(0) = 506$ GeV ^{4,5}. For the third-fourth two-quark-generation case we input the following $q^2/\Lambda_{QCD}^2 = 0$ values of the mixing angles for the up-sector and down-sector: $\theta_{up} = 0.5$ radians, and $\theta_{down} = \theta_{up} - \theta_{cabibbo}$ with $\theta_{cabibbo} = 0.22$ radians ⁵. We are primarily interested in the question of the running of the mixing angles.

Table 1. Two-quark-generation mass results, for $A = 5 \times 10^{-5}$, $B = 9.77 \times 10^8$.

quark	$M_q(A)$	$M_q(B)$	$\frac{M_{QCD+H}}{M_{QCD}}$
bottom	$27.2\Lambda_{QCD}$	$15.3\Lambda_{QCD}$	1
top	$993\Lambda_{QCD}$	$840\Lambda_{QCD}$	1.01
bottom'=top'	$2810\Lambda_{QCD}$	$3150\Lambda_{QCD}$	$\neq 1$, varies

The QCD plus Higgs boson case of the bottom quark mass function does not deviate significantly from the pure QCD running mass function. The QCD plus Higgs boson case of the top quark mass function shows some deviation from pure QCD. The Higgs boson term has an opposite sign relative to the gluon term, so it drives the top mass function up, in contrast to the usual QCD-only decreasing mass function. The Higgs boson has a substantial effect on the fourth generation quark self-energy. In fact, it causes the running fourth generation quark masses to increase in the asymptotic region. The results are summarized in the table, where A and B are the initial and final integration points.

By studying the mixing angle results, we determine that the running of the mixing angles due to any effects in our model is less than one part in 10^9 , for any quark mass input value up to and including 500 GeV. This is expected for pure QCD and can be proven analytically for QCD ⁶. We have been unable to prove analytically that the cabibbo angle is constant when the Yukawa interaction is included in the SDE analysis, but we have established numerically that there can be no observable effect even for the heavy hypothetical fourth generation.

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3. We choose a median value, $x_0 = 5$, in comparison with the $x_0 = 10$ value of Munczek and Jain ⁴, and the values $1 \geq x_0 \geq 7.3$ of D. Atkinson *et. al.* ⁷, for example.
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